A formal proof that Moore's proof is transmissive

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Draft of August 20, 2009

ABSTRACT

I argue that Moore's controversial proof of the existence of the world is justification-transmissive if epistemic justification is interpreted as incremental confirmation and some standard and intuitive assumptions are made. My argument crucially relies on Shogenji's theorem of transitivity of probabilistic support.

Moore's proof of the existence of a material world can be reproduced as follow:

e. My experience is that of a hand held up in front of my face.

p. Here is a real hand.

Therefore:

q. There is a material world.

p entails *q*, and *e* describes purported evidence for *p*. The sceptic about the existence of a material world contends that we possess no justification for believing *q*.[1] According to the sceptic, *e* does not justify belief in *q*. Those who use Moore's proof to respond to the sceptic concede that *e* supplies no *direct* justification for *q*, but they argue that *e* supplies *indirect* justification for *q*. They contend that *e* affords some justification for *p* and that this justification flows, through the entailment, to *q*.

Moore's proof is controversial for at least two reasons: concerning the question of whether *e* actually justifies *p* in the first instance, and concerning whether the justification for *p* from *e* (provided there is one) actually transmits across the entailment to *q*.[2] For example, while Pryor

[1] This is at least in one important version of this form of scepticism.

[2] Surely, there may be further points of controversy. For instance, supposing that the justification for *p* transmits to *q*, a question might be whether this justification suffices for the rational acceptance of *q*. I will not investigate these additional issues here.

(2004) answers 'yes' to both questions, Wright (2002) answers 'yes' to the first[3] but 'no' to the second.

In this paper I will make a case that *e* does justify *p* and that this justification does transmit to *q*. As arguments typically depend on some assumption, let me make my assumptions explicit right here. I will assume – among other things – the classic Bayesian framework according to which propositions have sharp probability values, the fashionable interpretation of epistemic justification as incremental confirmation, classical logic, and the intuitive principle that the evidential strength of a perception report *E* stating that *P* is the case is annulled if *P* is known to be false.

Suppose we decide to follow Okasha (2004) and White (2006) in formalizing epistemic justification into incremental confirmation, so that we stipulate that *E* justifies *P* just in case $\Pr(P|E) > \Pr(P)$,[4] where the probability function Pr expresses a rational agent's degree of confidence in a proposition. This formalization is not implausible and will appear natural to many.[5] If *this* is our notion of justification, then *e* does justify *p*. For it is strongly intuitive that $\Pr(p|e) > \Pr(p)$. If you are not convinced, consider that the probability that there is a hand here (or any alternative thing) is certainly minuscule; thus $\Pr(p) \cong 0$. On the other hand, it is very implausible that $\Pr(p|e) \cong 0$ – the value of $\Pr(p|e)$ cannot be minuscule. For if $\Pr(p|e) \cong 0$, given that $\Pr(p|e) = 1 - \Pr(\sim p|e)$, it should be true that $\Pr(\sim p|e) \cong 1$. This means that it is *practically certain* that if I perceive a hand, there is no hand at all. Even the staunchest sceptic should recoil at this extreme thesis. Hence $\Pr(p|e) > \Pr(p)$.

Now the question is whether the justification for *p* transmits to *q*. How can we formulate a principle of transmission of justification across entailment in Bayesian language? This is presumably the simplest formulation. The justification afforded by *E* for *P* transmits to *P*'s logical

[3] Wright seems to contend that *e* provides *sufficient* justification for believing *p only* if *q* is part of background information.
[4] In this paper I assume that the conditional probabilities like $\Pr(P|E)$ are well defined, in the sense that $\Pr(E) > 0$.
[5] Whether incremental confirmation reflects accurately the informal notion of epistemic justification is to some extent controversial. In this paper I will put aside these problems, and I assume the sceptic I envisage does the same.

consequence Q just in case this conditional is *non-vacuously* true (i.e. both its antecedent and its consequent are true):

(Transmission) If $\Pr(P|E) > \Pr(P)$ and P entails Q, then $\Pr(Q|E) > \Pr(Q)$.

We say that transmission *fails* – in the interesting sense – if (Transmission)'s antecedent is true but (Transmission)'s consequent is false. As a matter of fact, (Transmission) is known to be false for probability distributions such that $\Pr(P)$ and $\Pr(P|E) \in (0, 1)$.[6] Thus, though e, p and q verify (Transmission)'s antecedent, there is prima facie no guarantee that (Transmission)'s consequent is verified by them. Can we show that Moore's proof is just non-transmissive?

Consider the popular *variant* of Moore's proof in which q is replaced with q^* = 'I am not a handless brain in a vat having the computer-generated hallucination of a hand'. Clearly p entails q^*. As White (2006) has shown, this variant does falsify (Transmission). Briefly, if I knew that $\sim q^*$ (i.e. 'I am a handless brain in a vat that has the hallucination of a hand'), I could certainly *predict* that e. Thus $\sim q^*$ must make e very probable, which is otherwise not probable. Hence, $\Pr(e|\sim q^*) > \Pr(e)$. This trivially entails that $\Pr(\sim q^*|e) > \Pr(\sim q^*)$, which in turn implies that $\Pr(q^*|e) < \Pr(q^*)$. In conclusion, e justifies p, p entails q^*, but e does not justify q^*. A failure of transmission is instantiated. Unfortunately, this reasoning is of no help to establish that Moore's *original* proof – in which q is replaced back for q^* – is not transmissive. The reason is simple: my knowing that $\sim q$ ('there is no material world') does not allow me to predict that e. In other words, it turns out to be false that $\Pr(e|\sim q) > \Pr(e)$ in the first instance.

Besides White (2006)'s formalization, there are a quite few attempts to explain failure of transmission in Bayesian terms independently of Moore's proof. The most thorough formalizations have been put forward by Okasha 2004, Chandler 2009 and Moretti 2009. Unfortunately Okasha's formalization cannot apply to Moore's original proof. Its application would require that $\Pr(p|e) \leq \Pr(p)$, which is plainly false. On the other hand, Chandler's and Moretti's models can apply to

[6] As is well known, incremental confirmation does not satisfy Hempel's Special Consequence Condition.

Moore's argument, but only with the proviso that certain controversial epistemological assumptions are true.[7] Furthermore, Chandler and Moretti make use of formal notions of justification different from incremental confirmation.[8] In conclusion, we possess no demonstration that Moore's proof is not transmissive – at least if justification is interpreted as incremental confirmation. Indeed, I believe that if epistemic justification coincides with incremental confirmation, Moore's proof is transmissive.

On closer inspection, (Transmission) appears unsuitable to formalize the intuitive idea of transmission of justification. The intuition we want to capture is that – when justification transmits – E supports Q *indirectly*, via the mediation of P. (Transmission) does not capture this intuition. For the non-vacuous truth of (Transmission) appears compatible with the possibility that E justifies both Q and P simultaneously in a way that the justification for Q is not mediated by P.[9] A simple way to capture the idea that the justification for Q is mediated by P is to impose the additional constraint that were P known to be false, learning E would not provide support for Q; formally: $\Pr(Q|E \,\&\, {\sim}P) \leq \Pr(Q|{\sim}P)$. Let us then replace (Transmission) with:

(Transmission*) If $\Pr(P|E) > \Pr(P)$, $\Pr(Q|E \,\&\, {\sim}P) \leq \Pr(Q|{\sim}P)$ and P entails Q, then $\Pr(Q|E) > \Pr(Q)$.

We say now that the justification afforded by E for P transmits to P's logical consequence Q just in case (Transmission*) is non-vacuously true. We also stipulate that the justification from E for P fails to transmit to Q (in the interesting sense) just in case (Transmission*) is false.

We saw that the sceptic about the material world is very plausibly committed to claiming that $\Pr(p|e) > \Pr(p)$. Is the sceptic committed also to the claim that $\Pr(q|e \,\&\, {\sim}p) \leq \Pr(q|{\sim}p)$? I believe so. Suppose the sceptic accepts that $\Pr(q|e \,\&\, {\sim}p) > \Pr(q|{\sim}p)$. In that case, for her, e justifies q given

[7] Basically the assumption that the e supplies justification sufficient for rationally believing p only if q is already and independently justified.

[8] As they aim to model Wright's specific notion of evidential warrant.

[9] This is also noted by Chandler (2009).

$\sim p$. Given that either p or $\sim p$,[10] the sceptic is forced to conclude that either q is *true* (since p entails q) or q is *justified* by e. But this amounts to her capitulation! Saying that the sceptic is committed to accepting that $\Pr(q|e \& \sim p) \leq \Pr(q|\sim p)$ is not saying that this formula is true. I return to the issue of this truth-value below. For the moment note that those who appeal to Moore's proof to persuade the sceptic that p *indirectly* justifies q should most probably presuppose that $\Pr(q|e \& \sim p) \leq \Pr(q|\sim p)$.

(Transmission*) is false for some probability functions. Yet it proves *true* when its antecedent is satisfied such that $\Pr(Q|E \& \sim P) = \Pr(Q|\sim P)$ with $\Pr(Q) < 1$. For, in that case, the four conditions of Shogenji (2003)'s theorem of *transitivity of confirmation* are satisfied. The result is that $\Pr(Q|E) > \Pr(E)$. Shogenji shows that, given three propositions E, P and Q, if

(1) $\Pr(P|E) > \Pr(E)$,

(2) $\Pr(Q|P) > \Pr(Q)$,

and P *screens off* E *with respect to* Q, that is,

(3) $\Pr(Q|E \& P) = \Pr(Q|P)$,

(4) $\Pr(Q|E \& \sim P) = \Pr(Q|\sim P)$,

then $\Pr(Q|E) > \Pr(Q)$.[11]

The sceptic is certainly committed to the claim that $\Pr(q) < 1$. Is the sceptic also committed to the claim that $\Pr(q|e \& \sim p) = \Pr(q|\sim p)$? Shogenji (2003) emphasizes that when E reports perceptual evidence that P, the screening off conditions (3) and (4) appear in general true. For 'once the truth/falsity of P is given, it is unreasonable to let … the (apparent) perception that P affect the probability of Q, even if … that (apparent) perception would otherwise affect the probability of Q' (615-16). This strikes me as straightforward. Since e does report perceptual evidence that p, the sceptic is prima facie committed to the claim that $\Pr(q|e \& \sim p) = \Pr(q|\sim p)$. The sceptic could rebuff this commitment if she could make a convincing case that for the specific triple of e, p and q it is

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[10] I am assuming that the sceptic about the material world accepts classic logic. I see no compelling reason why she should reject it.

[11] If (Transmission*)'s antecedent is true, (1) is true. Furthermore, since P entails Q, $\Pr(Q|E \& P) = \Pr(Q|P) = 1$ and $\Pr(Q|P) = 1$, thus (3) is true and, if $\Pr(Q) < 1$, (2) is true too. Hence if (4) $\Pr(Q|E \& \sim P) = \Pr(Q|\sim P)$, it follows that $\Pr(Q|E) > \Pr(E)$.

true, or at least non-implausible, that $\Pr(q|e \;\&\; {\sim}p) < \Pr(q|{\sim}p)$. But what reason could the sceptic adduce?

Shogenji (2003: 15, note 4) recognizes that there may be 'degenerate cases' of triples of propositions *E*, *P* and *Q*, where *E* describes perceptual evidence that *P*, such that they satisfy (1) and (2) but do not satisfy the screening off conditions (3) and (4). In these cases, *E* does *not* serve as perceptual evidence when occurs in the inequalities that replace (3) and (4). Namely, the fact that *E* supports *Q* conditional on *P* or ~*P* does not derive from the fact that *E* supports its representational content conditional on *P* or ~*P* (which is impossible), but from independent features of the specific epistemic context. Here is an example: consider again the variant of Moore's proof in which *q* is replaced with *q**. In this case it is presumably true that $\Pr(q^*|e \;\&\; {\sim}p) < \Pr(q^*|{\sim}p)$. For the conjunction of *e* and ~*p* ('I have the apparent perception of a hand and there is no hand') is evidence *more specific* than ~*p* alone for the hypothesis that ~*q* ('I am a handless brain in a vat having the hallucination of a hand'). Hence $\Pr({\sim}q^*|e \;\&\; {\sim}p) > \Pr({\sim}q^*|{\sim}p)$, which entails that $\Pr(q^*|e \;\&\; {\sim}p) < \Pr(q^*|{\sim}p)$.

Yet when *q* is replaced back for *q**, it is hard to find any grounds for the thesis that $\Pr(q|e \;\&\; {\sim}p) < \Pr(q|{\sim}p)$. There is no logical or epistemic feature I can see in the triple of *e*, *p* and *q* that could entitle one to accept the above inequality or even to judge it not to be implausible. Since *e* reports perceptual evidence that *p*, the sceptic should stick to the intuitive and natural claim that $\Pr(q|e \;\&\; {\sim}p) = \Pr(q|{\sim}p)$ on pain of making arbitrary or ad hoc assumptions. Hence, the sceptic should derive that $\Pr(q|e) > \Pr(q)$. This is why Moore's proof is transmissive.[12]

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[12] I am grateful to Branden Fitelson and Tomoji Shogenji for comments and criticism on previous drafts of this paper.

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